sQ_1-DEGREES OF COMPUTABLY ENUMERABLE SETS

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Tennenbaum (see [3, p.159]) defined the notion of Q-reducibility on stes of natural numbers as follows: a set A is Q-*redusible* to a set B (in symbols: $\mathrm{A} \leq_{\mathrm{Q}} \mathrm{B}$) if there exists a computable function f such that for every $x \epsilon \omega$ (where ω denotes the set of natural numbers),

$$x \epsilon A \iff W_{f(x)} \subseteq B.$$

We say in this case that $\mathrm{A} \leq_{\mathrm{Q}} \mathrm{B}$ via f. If $\mathrm{A} \leq_{\mathrm{Q}} \mathrm{B}$ via a computable function f and there exists a computable function g such that for all x, y,

$$y \epsilon W_{f(x)} \Rightarrow y \leq g(x),$$

then we say that A is sQ-*reducible* to B (in symbols: $\mathrm{A} \leq_{\mathrm{sQ}} \mathrm{B}$) via f *and* g (see [1]). If $\mathrm{A} \leq_{\mathrm{sQ}} \mathrm{B}$

and for all x, y,

$$x \neq y \Rightarrow W_{f(x)} \cap W_{f(y)} = \emptyset,$$

then we say that A is sQ_1–*reducible* to B (in symbols: $\mathrm{A} \leq_{\mathrm{sQ}_1} \mathrm{B}$) *via* f *and* g .

Our notation and terminology are standard and can be found in [2, 3].

We prove that there exist two computably enumerable (c.e.) sets having no least upper bound on the sQ_1 –reducibility ordering. We show that the c.e. sQ_1 –degree are not dense. We prove that if **a** is a c.e. sQ_1 –degree such that

$$\mathbf{0}_{sQ_1} <_{sQ_1} \mathbf{a} <_{sQ_1} \mathbf{0}'_{sQ_1},$$

then there exist infinitely many pairwise sQ -incomparable c.e. sQ -degrees $\{c_i\}_{i\epsilon\omega}$ such that for all i,

$$\boldsymbol{a}_{sQ_1} <_{sQ_1} \boldsymbol{c_i} <_{sQ_1} \mathbf{0}'_{sQ_1}.$$

References

[1] R.Sh.Omanadze, On the upper semilattice of recursively enumerable sQ-degrees, Algebra and Logic , 30, 4 (1993) 265-271.

[2] H.Rogers, Theory of recursive functions and effective computability. McGraw-Hill Book Co., New York, 1967.

[3] R.I.Soare, Recursively Enumerable Sets and Degrees. Springer, 1987.