On an optimization problem of the delay parameter containing in controls

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Let $t_1 > t_0$ and $\theta_2 > \theta_1 > 0$ be given numbers; let $O \subset R^n$ be an open set and let $U \subset R^{r_1}$ and $V \subset R^{r_{21}}$ be convex and compact sets. Furthermore, let $x_0 \in O$ be fixed point and let the n-dimensional function $f(t, x, u, u_1, v, v_1)$ be continuous on $[t_0, t_1] \times O \times U^2 \times V^2$ and continuously differentiable with respect to (x, u, u_1, v, v_1). Let Ω_1 be a set of piecewise-continuous control functions $u(t) \in U$, $t \in I = [t_0 - \theta_2, t_1]$ and Ω_2 be a set of absolutly continuous control functions $v(t) \in V$, $t \in I$. To each element $w = (\theta, u(\cdot), v(\cdot)) \in W = [\theta_1, \theta_2] \times \Omega_1 \times \Omega_2$ we assign the ordinary differential equation

$$\dot{x} = f(t, x, u(t), u(t-h), v(t), v(t-\theta)),\ t \in [t_0, t_1] \qquad (1)$$

with the initial condition

$$x(t_0) = x_0\ , \qquad (2)$$

where $h \in [\theta_1, \theta_2]$ is a fixed delay parameter.

Definition 1. Let $w \in W$, a function $x(t) = x(t; w) \in O, t \in [t_0, t_1]$, is called a solution of equation (1) with the initial condition (2) or a solution corresponding to the element w if it satisfies condition (2) and is absolutely continuous and satisfies equation (1) almost everyweher.

Let the scalar-valued functions $\vartheta^i(\theta, x), i = \overline{0, l}$, be continuously differentiable on $[\theta_1, \theta_2] \times O$.

Definition 2. An element $w \in W$ is said to be admissible if

$$\vartheta^i(\theta, x(t_1; w)) = 0,\ i = \overline{1, l}. \qquad (3)$$

Denote by W_0 the set of admissible elements.

Definition 3. An element $w_0 = (\theta_0, u_0(\cdot), v_0(\cdot)) \in W_0$ is said to be optimal if for an arbitrary element $w \in W_0$ we have

$$\vartheta^0(\theta_0, x_0(t_1)) \le \vartheta^0(\theta, x(t_1)),\ \ x_0(t) = x(t; w_0). \qquad (4)$$

For the delay optimization problem (1)-(4) the necessary conditions of optimality are obtained on the basis of variation formulas [1] and by the scheme given in [2].

References

[1] Iordanishvili M. Local variation formulas of solutions for the nonlinear controlled differential equation with the discontinuous initial condition and with delay in the phase coordinates and controls. *Transactions of A. Razmadze Mathematical Institute*, **173** (2) (2019), 10-16.

[2] Tadumadze T. Variation formulas of solutions for functional differential equations with several constant delays and their applications in optimal control problems. *Mem. Differential Equations Math. Phys.*, **70** (2017), 7-97.