Calculations of Berry's connection and Wilson loop for topological insulators (part 1)

***Merab Eliashvili*, Giorgi Tsitsishvili**

E-mail: merab.eliashvili@tsu.ge
Department of Physics, Faculty of Exact and Natural Sciences,
Ivane Javakhishvili Tbilisi State University
3, I. Chavchavadze Ave., Tbilisi, 0179, Georgia

- A topological insulator is a bulk insulator which is conducting at the boundary with another material or with vacuum. Topological insulator is characterized by topological quantum numbers -so called topological invariants
- Topological invariant is a quantity which is constant under continuous deformations of quantum Hamiltonian. These invariants are defined from the energy spectrum. They are identified as Chern number of a vector bundle over the Brillouin zone. Chern numbers are calculated using Berry's connection in the space of Bloch wave functions.
- We report on the calculations of Berry's phase using eigen values and eigen states of the $N \times N$ Hermitian matrix Hamiltonian $\mathcal{H}(\boldsymbol{\kappa})$
$$\mathcal{H}(\boldsymbol{\kappa})\phi_n(\boldsymbol{\kappa}) = E_n(\boldsymbol{\kappa})\phi_n(\boldsymbol{\kappa}); \qquad n = 1,2,\dots,N$$
Matrix-valued connection is defined by
$$[A_\mu]_{mn} = i\langle\phi_n(\boldsymbol{\kappa})|\nabla_\mu|\phi_m(\boldsymbol{\kappa})\rangle$$
- Corresponding curvature is given by
$$[F_{\mu\nu}]_{mn} = -i\langle\phi_n(\boldsymbol{\kappa})|\nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu|\phi_m(\boldsymbol{\kappa})\rangle$$
- Singular points of curvature $\boldsymbol{\kappa}_0$
$$[F_{\mu\nu}(\boldsymbol{\kappa})]_{mn} = 0 \qquad \boldsymbol{\kappa} \neq \boldsymbol{\kappa}_0$$
Are identified calculating the Wilson loop operator
$$W(\boldsymbol{\kappa}_0) = Tr\left\{P\left[i\oint_\gamma d\,\boldsymbol{\kappa}\cdot\boldsymbol{A}(\boldsymbol{\kappa})\right]\right\}$$
- In the part 1 of the report we give basic definitions and state the problem. In the part 2 main points of calculations will be given.