On the stochastic derivative of the Poisson functional

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In the theory of stochastic integration, in contrast to the standard integration theory, besides the fact that the integrand is the measurable function of two variables, it should be the adapted (nonanticipated) process, i. e. it should be independent of the future increments of the Wiener process. Skorokhod (1975) replaced this requirement with the requirement of smoothness in some sense of the integrand. Later this idea was developed in the works of Gaveau-Trauber (1982), Nualart, Zakai (1986), Pardoux (1982), Protter, Malliavin (1979), etc. In particular, Gaveau-Trauber have proved that the Skorokhod operator of stochastic integration coincides with the conjugate operator of a stochastic derivative (with the so-called Malliavin's) operator.

It should be noted that in modern stochastic analysis special place take the so-called martingale representation theorems, which implies the representation of the adapted functionals in the form of stochastic integrals. Ocone (1984) proved that the integrand of the above-mentioned stochastic integral coincides with the predictable projection of the stochastic derivative of the functional. It is known that in the Weiner case there are two equivalent definitions of a stochastic derivative, but in general, for so called normal martingale classes (which include compensated Poisson functionals) these definitions are not equivalent. Ma, Protter, Martin (1998), built the corresponding example.

In the present work, a new constructive definition of the stochastic derivative of the polynomial Poisson functional is introduced. It is shown it is equivalent to a general definition based on a chaotic expansion of functional, and its properties are studied. The stochastic integral representation theorem with an explicit expression of the integrand is proved.